Precise Calculations and Measurements on the Complex Dielectric Constant of Lossy Materials Using TM_{010} Cavity Perturbation Techniques

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Abstract—An exact field theory solution for the cylindrical TM_{010} cavity with a coaxial lossy dielectric cylinder is given. The error in the calculated field solutions is estimated to be less than 1 percent of the true values. Correction for the cavity holes used to introduce the sample is taken into account. The exact solution shows that the real part of the permittivity (ϵ') is a complex function of both the frequency shift and the change in the loaded Q-factor (Q_L). On the other hand the imaginary part (ϵ'') is nearly proportional to $\delta(1/Q_L)$ and it has different slopes for varying frequencies.

By means of active cavity techniques already reported [1], experimental measurements on ϵ' and ϵ'' taken at 2.2 GHz on a number of materials (water, teflon, n-proponal, methanol, etc.) agree with published data within 1 percent even when using large samples.

I. Introduction

Circular TM_{010} cavities have been widely used in microwave measurements of dielectric materials since 1946 [2], and further detailed studies [3], [4] for plasma diagnostic purposes were carried out in the 1960's. This method of measurement is still of considerable interest due to the great accuracy (1 percent) derived from new techniques using simple real time automatic frequency measurements as a means of measuring the resonant frequency shift and the loaded Q-factor (Q_L) variation [1]. As dielectric measurements become more automated it is important to reduce the requirements on sample geometry and the allowed range of dielectric values by developing a theory suitable to interpret experimental data over a wide range of sample sizes, dielectric properties, and cavity sample insertion holes.

In recent years, the measurement of permittivity of lossy materials has been becoming more and more important. Successful work has been reported on the subject of a transmission line measurement method based on the measurement of impedance [5], [6] or open-ended cavity [7], [8]. Every kind of impedance measurement technique including the automatic network analyzer has been widely used in the field of wide-band dielectric permittivity measurements. But, in the published works on the cylindrical TM_{010} cavity perturbation theory only low loss materials were taken into account [4], [9], [10]. In this paper we present an exact theory using field matching techniques accurate to within 1 part in 10^3. A dispersion equation is obtained which allows for large values in both ϵ' and ϵ''. The digital computation is done on an IBM 360 computer, with a CPU time of less than 1 s for calculating a pair of values of ϵ' and ϵ''. Correction for the presence of a cavity opening for introducing the sample material is allowed for in the exact theory. Experimental results are taken and used to calculated the complex dielectric constant of a variety of materials. These values are then compared with published data.

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II. Theory

A. Simple Perturbation Theory

Mathematical complexities are normally avoided by the use of the simple perturbation theory which can be expressed for Fig. 1(a) as follows:

$$\epsilon_p' - 1 = 2J_1^2(x_{01})\frac{\delta f}{f_0}R_0^2/R_1^2 \tag{1}$$

$$\epsilon_p'' = J_1^2(x_{01})\delta(1/Q_L)R_0^2/R_1^2 \tag{2}$$

where

$$\delta f = f_0 - f_S \tag{3}$$

$$\delta(1/Q_L) = 1/Q_{LS} - 1/Q_{L0} \tag{4}$$

f_0	resonance frequency of empty cavity;
f_S	resonance frequency with test sample;
Q_{L0}	the loaded cavity Q-factor of empty cavity;
Q_{LS}	the loaded cavity Q-factor of cavity with test sample;
R_0	radius of cavity;
R_1	radius of sample;
x_{01}	is the first root of $J_0(x)=0$; and
ϵ_p' and ϵ_p''	respectively, real and imaginary parts of the complex dielectric constant calculated by the simple perturbation theory.

B. Exact Theory (Lossless Material)

For larger samples, or samples with larger dielectric constants, a dispersion equation has already been obtained

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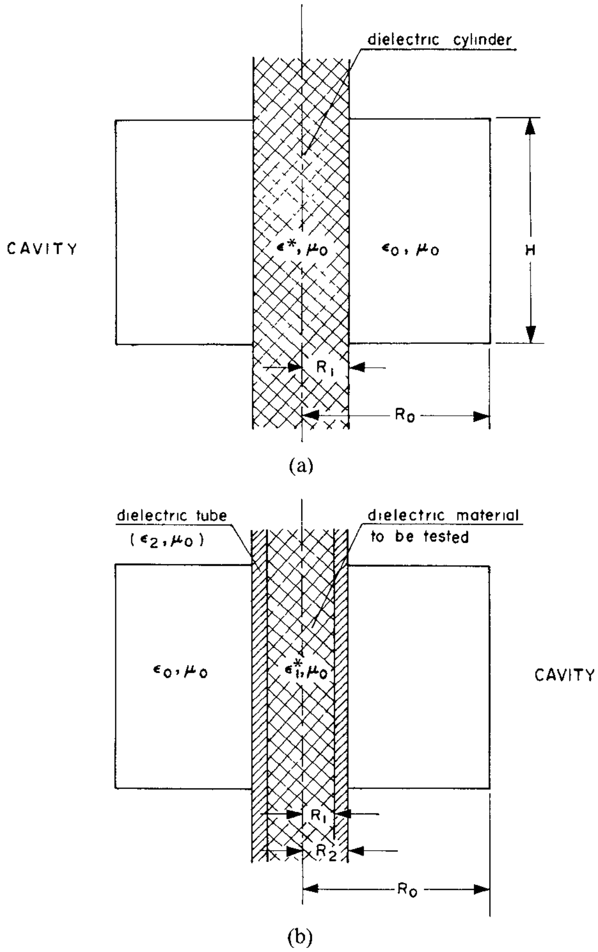


Fig. 1. The cylindrical TM_{010} cavity with coaxial lossy dielectric materials. (a) Single solid dielectric. (b) Liquid or gas test material inside a dielectric tube.

under the conditions of low loss. For the case illustrated in Fig. 1(b) the following expression is available [4]:

$$\frac{k_1 J_1(k_1 R_1)}{J_0(k_1 R_1)} = F(k_0, \epsilon_2, R_1, R_2, R_0) \tag{5}$$

where

$$F = \sqrt{\epsilon_2}\, k_0 \frac{J_{121}Y_{022} - Y_{121}J_{022} - A(J_{121}Y_{122} - Y_{121}J_{122})}{J_{021}Y_{022} - Y_{021}J_{022} - A(J_{021}Y_{122} - Y_{021}J_{122})} \tag{6.1}$$

$$A = \sqrt{\epsilon_2}\, \frac{J_{000}Y_{002} - J_{002}Y_{000}}{J_{000}Y_{102} - J_{102}Y_{000}} \tag{6.2}$$

$$k_0 = 2\pi f_S \sqrt{\epsilon_0 \mu_0} \tag{6.3}$$

$$k_1 = \sqrt{\epsilon_1}\, k_0 \tag{6.4}$$

$$k_2 = \sqrt{\epsilon_2}\, k_0. \tag{6.5}$$

J_{npq} and Y_{npq} express $J_n(k_p R_q)$ and $Y_n(k_p R_q)$, the Bessel functions of the first kind and second kind, respectively, where

$$n = 0 \text{ or } 1$$
$$p = 0 \text{ or } 2$$
$$q = 0, 1 \text{ or } 2$$

ϵ_1 and ϵ_2 are the relative permittivities of the materials in region 1 and 2, ϵ_0, μ_0 are respectively the permittivity and permeability of free space, and f_S is the resonant frequency of the cavity with the material.

The corresponding dispersion equation for case (a) in Fig. 1 is obtained by allowing R_2 to approach R_1 and by letting $\epsilon_2 = \epsilon_1$. This case corresponds to the use of solid dielectric materials and the simplified expression for (6.1) is as follows:

$$F(k_0, R_1, R_0) = k_0 \frac{J_{101}Y_{000} - J_{000}Y_{101}}{J_{001}Y_{000} - J_{000}Y_{001}}. \tag{7}$$

Equations (5)–(7) provide an exact dependence between ϵ and f_S for lossless materials. It will later be shown that the above equations are still rather accurate for $tg\delta < 0.5$ but a different dispersion equation must be used for cases where $tg\delta > 0.5$. The high loss case is very important in dielectric measurements on biological solutions where both high dielectric losses and high dielectric constants can exist simultaneously.

C. Generalized Solution (Lossy Materials)

Only E_z and H_ϕ fields exist in a cylindrical cavity resonant in the TM_{0n0} mode. We can write the wave equation, valid for any value of ϵ^*, as follows:

$$\left(r\frac{d^2}{dr^2} + \frac{d}{dr} + k_1^2 r \right) E_z = 0 \tag{8}$$

where

$$k_1^2 = k^2 \epsilon^* = \alpha^2 - j\beta^2 \tag{8.1}$$

$$k^2 = k_0^2 \left(1 - \frac{1}{2Q_L^2} + j\frac{1}{Q_L} \right) \tag{8.2}$$

$$\epsilon^* = \epsilon'(1 - j tg\delta) \tag{8.3}$$

$$\alpha^2 = k_0^2 \epsilon' \left(1 - \frac{1}{2Q_L^2} + \frac{1}{Q_L} tg\delta \right) \tag{8.4}$$

and

$$\beta^2 = k_0^2 \epsilon' \left[\left(1 - \frac{1}{2Q_L^2} \right) tg\delta - \frac{1}{Q_L} \right]. \tag{8.5}$$

The solution of the wave equation (8) is known to be in the form of Bessel functions when k_1 is real. However under consideration of lossy media, k_1 is complex. We can take the series form solution as follows:

$$E_z = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} \left[\frac{k_1 r}{2} \right]^{2n}. \tag{9}$$

By substituting (8.1)–(8.5) into (9) and separating into real and imaginary parts we find

$$E_z = E_z' + jE_z'' \tag{10}$$

$$E_z' = J_0(\alpha r) - \frac{\beta^4 r^2}{8\alpha^2} J_2(\alpha r) + \frac{\beta^8 r^4}{384\alpha^4} J_4(\alpha r) \cdots \tag{10.1}$$

and

$$E_z'' = \frac{r\beta^2}{2\alpha}\left[J_1(\alpha r) - \frac{\beta^4 r^2}{24\alpha^2}J_3(\alpha r) + \frac{\beta^8 r^4}{1920\alpha^4}J_5(\alpha r)\cdots\right]. \tag{10.2}$$

Both equations (10.1) and (10.2) are rapidly convergent series and, if only the first two terms are taken, the relative error is smaller than 1 part in 10^3 whenever $\epsilon'' < (\lambda_0/\pi R_1)^2$. The value of λ_0 is equal to the free space wavelength at resonance. The radius of the sample is in general smaller than one twentieth of λ_0 (e.g., smaller than 5.0 mm at 3 GHz) so that the first two terms of (10.1) and (10.2) are sufficient even if ϵ'' is as high as 50.

It should also be noted that a similar method for solving other geometries (rectangular, spherical, etc.) may be employed. If the imaginary part of the argument of Bessel functions is small (e.g., $|k_1'' r| \leq 0.1|k_1' r|$, where $k_1 = k_1' + jk_1''$) we can take the Taylor's series expansion as follows:

$$B_n(k_1 r) = B_n(k_1' r) + jk_1'' r B_n'(k_1' r) + \cdots. \tag{11}$$

It is easily shown that (11) is an approximate expression of (10) except in the vicinity of $B_n(k_1' r) = 0$.

The corresponding azimuthal magnetic field may then be calculated from the following expression:

$$H_\phi = -\frac{1}{j\omega\mu_0}\frac{\partial E_z}{\partial r}. \tag{12}$$

D. The Dispersion Equation (Exact Theory)

In the first case we assume a cavity made with a low-loss conductor (e.g. copper, brass, aluminum, etc.) using a dielectric tube (quartz, pyrex, etc.) to conduct a high-loss liquid or gas through the cavity. In this case the Q_L of the cavity, with sample, will be mainly determined by the lossy test sample. The electrical field in the test material can then be expressed by (10.1) and (10.2). The same field outside the sample must be a linear combination of the first and second type of Bessel functions of order zero. Every Bessel function is expressed with a complex argument in its Taylor series expansion as shown in (11), and the value of $J_0(k_0 R)$ is nearly equal to zero. Then by the use of field matching methods along the cylindrical surface of radius R_1 the following dispersion equation is obtained:

$$-\left.\frac{dE_z}{E_z dr}\right|_{r=R_1} = F^*(k, \epsilon_2, R_1, R_2, R_0) \tag{13}$$

where E_z is given by (10.1) and (10.2) and the complex function F^* is given as follows for cases (a) and (b), respectively:

$$F_a^* = k\frac{Y_{101} + j\eta_1 Y_{101}' - B^*(J_{101} + j\eta_1 J_{101}')}{Y_{001} - j\eta_1 Y_{101} - B^*(J_{001} - j\eta_1 J_{101})} \tag{14.1}$$

$$F_b^* = \sqrt{\epsilon_2}\, k\frac{F_1^* - A^* F_2^*}{F_3^* - A^* F_4^*} \tag{14.2}$$

where

$$\eta_n = \frac{k_0 R_n}{2}\left(\frac{1}{Q_{LS}} - \frac{1}{Q_{L0}}\right) \tag{14.3}$$

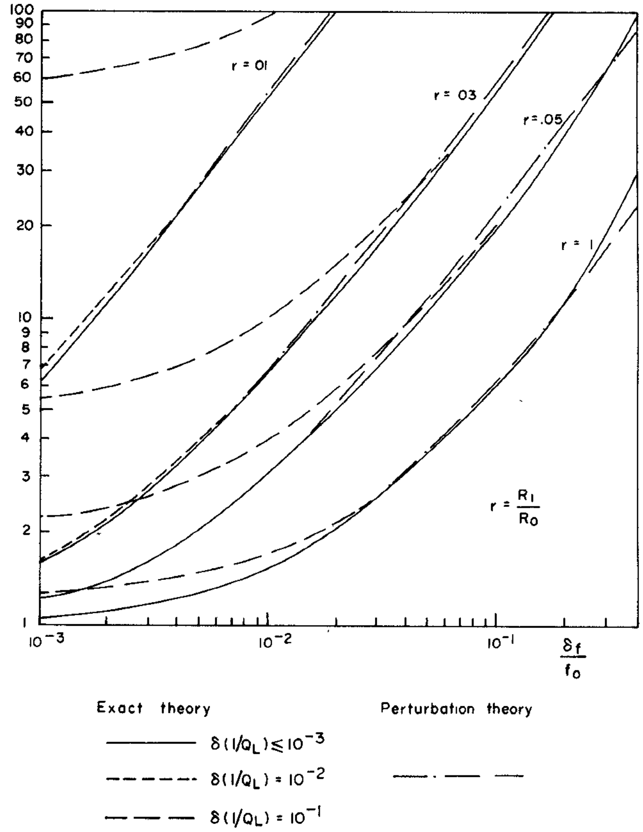


Fig. 2. The real part of the permittivity as a function of the resonance frequency shift and the loaded Q-factor for case of Fig. 1(a) using perturbation theory and exact theory.

Q_{LS} and Q_{L0} are the Q_L factor of cavity with dielectric sample and empty cavity, respectively,

$$B^* = \frac{Y_{000} - j\eta_0 Y_{100}}{J_{000} - \frac{1}{2}\eta_0^2 J_{200} - j\eta_0 J_{100}} \tag{14.4}$$

$$A^* = \sqrt{\epsilon_2}\,\frac{Y_{002} - j\eta_2 Y_{102} - B^*(J_{002} - j\eta_2 J_{102})}{Y_{102} + j\eta_2 Y_{102}' - B^*(J_{102} + j\eta_2 J_{102}')} \tag{14.5}$$

$$F_1^* = J_{121}Y_{022} - Y_{121}J_{022} + j[\zeta_1(Y_{022}J_{121}' - J_{022}Y_{121}') + \zeta_2(Y_{121}J_{122} - J_{121}Y_{122})] \tag{14.6}$$

$$F_2^* = J_{121}Y_{122} - Y_{121}J_{122} + j[\zeta_1(J_{121}'Y_{122} - J_{122}Y_{121}') + \zeta_2(Y_{122}'J_{121} - J_{122}'Y_{121})] \tag{14.7}$$

$$F_3^* = J_{021}Y_{022} - Y_{021}J_{022} + j[\zeta_1(Y_{121}J_{022} - J_{121}Y_{022}) + \zeta_2(J_{122}Y_{021} - Y_{122}J_{021})] \tag{14.8}$$

$$F_4^* = J_{021}Y_{122} - Y_{021}J_{122} + j[\zeta_1(J_{122}Y_{121} - J_{121}Y_{122}) + \zeta_2(J_{021}Y_{122}' - Y_{021}J_{122}')] \tag{14.9}$$

$$\zeta_n = \sqrt{\epsilon_2'}\,\eta_n \tag{14.10}$$

$$\epsilon_2 = \epsilon_2' - j\epsilon_2'' \tag{14.11}$$

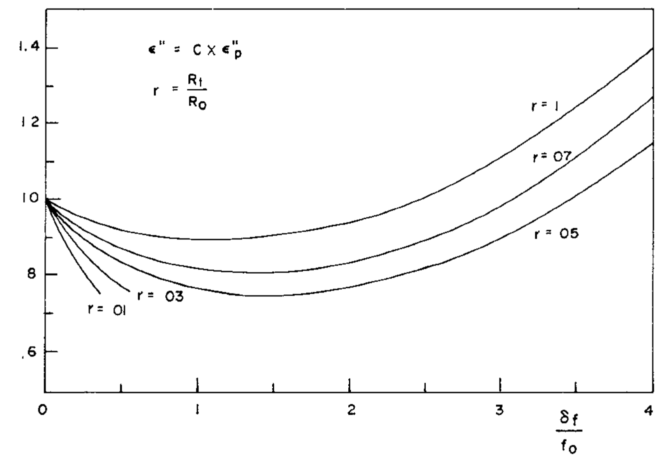


Fig. 3. A correction coefficient C relating the exact loss factor (ϵ'') to the simple perturbation loss factor (ϵ_p'') as per Fig. 1(a).

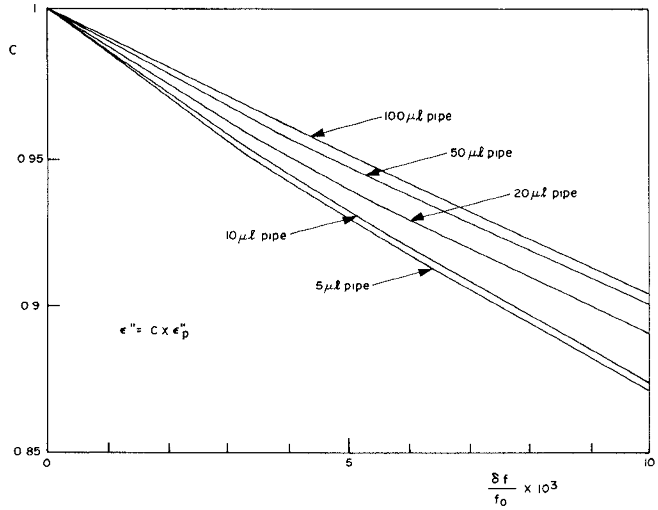


Fig. 4. The correction factor C versus the resonant frequency shift as per Fig. 1(b). The diameter of the cavity is 102.63 mm and a pyrex sampling pipe ($\epsilon'=4.75$, $\epsilon''=0.04$) is used.

and

$$k=k_0\left[1+j\frac{1}{2}\left(\frac{1}{Q_{LS}}-\frac{1}{Q_{L0}}\right)\right] \tag{14.12}$$

J_{npq} and Y_{npq} express $J_n(\sqrt{\epsilon_p}\,k_0R_q)$ and $Y_n(\sqrt{\epsilon_p}\,k_0R_q)$, respectively, and J_{1pq}' and Y_{1pq}' are the derivatives of J_{1pq} and Y_{1pq}, respectively.

In Fig. 2 the value of ϵ' is plotted as a function of $(\delta f/f_0)$ using both the perturbation theory and the exact theory for different values of Q_L. It is seen that the two theories produce very nearly the same results in most practical cases (($\delta f/f_0)\leqslant 0.01$, $\epsilon'\leqslant 10$ and for low-loss materials). The effects of Q_L on ϵ' is only apparent in the situation of low Q_L ($\leqslant 10^3$). It must be noted that a minimum Q-factor (for example 100) is necessary [1] for practical measurements. For measuring lossy materials, a smaller sample must be used, and even in such cases, the simple perturbation theory is still not suitable to interpret the experimental results. If the problem of low Q_L measurement ($Q_L<100$) were solved perfectly, the limitation of sample size would not be necessary and the reported theory would be more useful.

In the case of small losses ($tg\delta<0.5$) the use of simple equation (5) is accurate to within 1 percent. For the exact solution the imaginary part of the permittivity is also nearly always proportional to $\delta(1/Q_L)$. This property allows us to introduce a correction factor C for the case of Fig. 1(a) and Fig. 1(b) as shown in Figs. 3 and 4. This C factor is accurate to within 2 percent for $tg\delta\leqslant 1$ and to within 4 percent for $tg\delta\leqslant 4$.

Fig. 5 shows the value of ϵ' as a function of $\delta f/f_0$ in a series of practical cases in which a pyrex sampling pipe is used to introduce the test material into the cavity as shown in Fig. 1(b). Again the perturbation theory is sufficiently accurate in many practical cases ($\epsilon'\leqslant 30$ and $Q_L>10^3$). However, for lossy materials, the ϵ' must be determined by both $\delta f/f_0$ and $\delta(1/Q_L)$ using the exact theory.

E. Cavity Opening for Test Sample Insertion

According to the well known cavity theory, the loss in an empty cavity can be expressed as follows:

$$\frac{1}{Q_{L0}}=\frac{1}{Q_{W0}}+\frac{1}{Q_{EX0}} \tag{15}$$

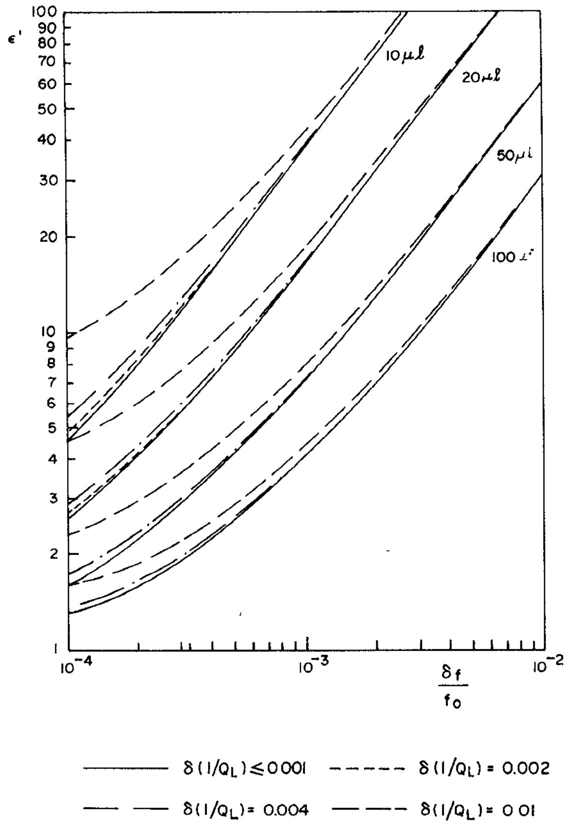


Fig. 5. The real part of permittivity versus the resonant frequency shift for Fig. 1(b). The diameter of the cavity is 102.63 mm ($=2R_0$) and the following sampling pipes ($\epsilon_2'=4.75$, $\epsilon_2''=0.04$) are used: 100 μl ($R_1=0.684$ mm; $R_2=0.952$ mm), 50 μl ($R_1=0.483$ mm; $R_2=0.845$ mm), 20 μl ($R_1=0.304$ mm; $R_2=0.749$ mm), 10 μl ($R_1=0.198$ mm; $R_2=0.711$ mm), and 5 μl ($R_1=0.152$ mm; $R_2=0.705$ mm).

where Q_{W0} and Q_{EX0} are the Q-factor contributed, respectively, by resistive ohmic losses in cavity walls and external circuits. After the sample is inserted, Q_{L0} is changed into Q_{LS}, and

$$\frac{1}{Q_{LS}}=\frac{1}{Q_W}+\frac{1}{Q_{EX}}+\frac{1}{Q_S}+\frac{1}{Q_h} \tag{16}$$

where Q_S and Q_h are the contribution of dielectric losses from sample and sample insertion hole, respectively. Q_W and Q_{EX} are the new values for the wall and external losses produced by the new field distributions in the cavity in the presence of the sample material.

For an exact measurement we must find the loss in the sample ($1/Q_S$). It is therefore very important to understand the variation of Q_W and Q_{EX} and to calculate the effects of the sample insertion hole. For this purpose, the perturbation theory is useful, and it has enough accuracy because all of these effects are very small (ordinarily a few percent in both $\delta f/f_0$ and $\delta(1/Q)$ measurement).

By the use of ordinary cavity wall perturbation procedures, one can calculate the ohmic losses in the cavity wall as follows:

$$L_w=\frac{R_s}{2}\int_s H_t^2\,dS \tag{17}$$

where R_s is the surface resistivity of cavity wall, and H_t is the tangential magnetic field at the cavity wall. Then, an explicit expression to describe the variation of Q_W can be obtained:

$$\frac{1}{Q_W}=\frac{1}{Q_{W0}}\left[1-\frac{(\epsilon'-1)r_1^2}{2J_1^2(x_{01})}\right] \tag{18}$$

where $r_1=R_1/R_0$ is the relative radius of sample.

The power coupled into the external circuits must be proportional to the square of the intensity of the RF fields where the coupling element is located. Based on this assumption it can be shown that

$$\frac{1}{Q_{EX}}=\frac{1}{Q_{EX0}}\left[1-\frac{3(\epsilon'-1)r_1^2}{2J_1^2(x_{01})}\right]. \tag{19}$$

A second major perturbation by the sample is produced at the sample insertion hole. Some authors have already discussed this problem [9], [10], but only the lowest TM_{01} mode was taken into account. By the use of the same procedure, but calculating the mode's amplitude by means of the Galerkin method [12], the resonant frequency shift and the additional loss produced by the sample insertion hole is obtained [11]. The resonant frequency deviation contributed by losses is given by

$$\frac{\delta f}{f_0}=\frac{1}{2}\delta\left(\frac{1}{Q_L}\right). \tag{20}$$

By the use of perturbation theory a pair of correction equations are obtained for the measurement of a lossy

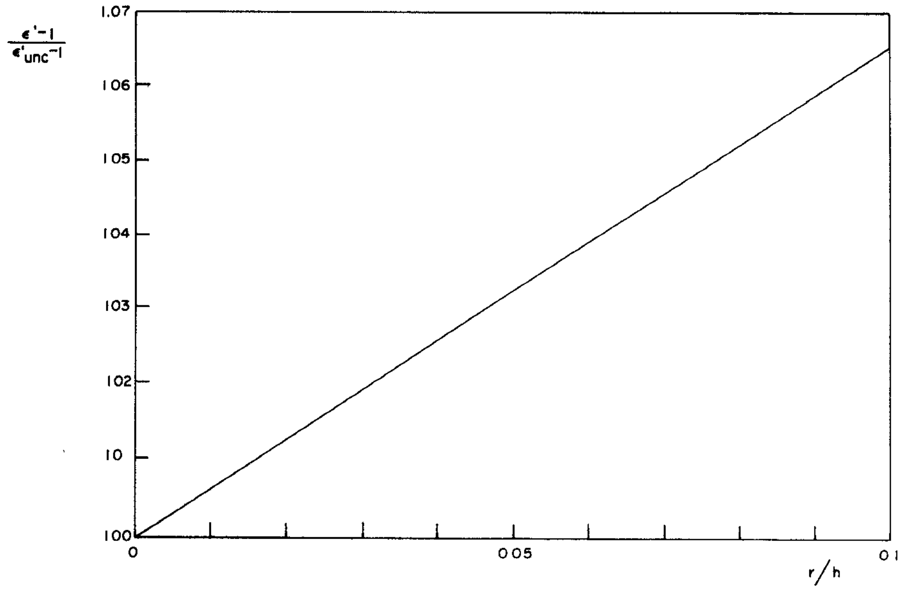


Fig. 6. The correction factor of ϵ' due to the sample insertion holes (dual holes in a TM_{010} cavity).

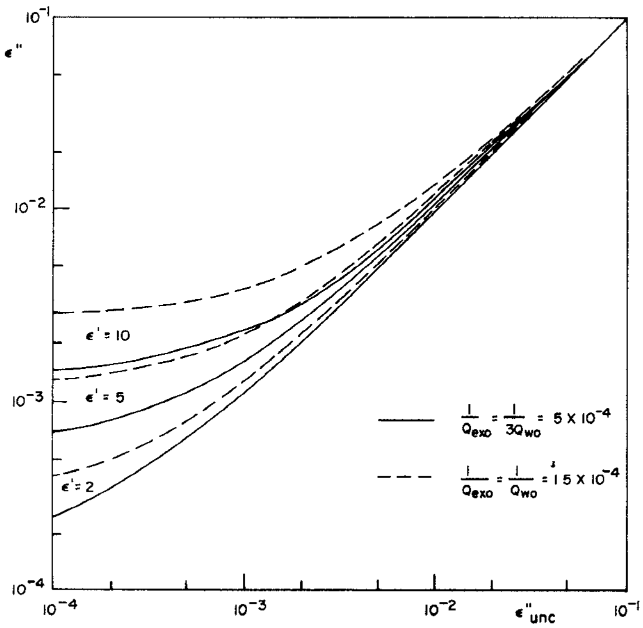


Fig. 7. The correction curve of ϵ'' due to sample insertion holes, the variation in cavity wall losses and coupling with external circuit (cavity size: $R_0=51.31$ mm; $H=40$ mm).

dielectric:

$$\epsilon'-1=(\epsilon'_{\mathrm{unc}}-1)\cdot\left\{1+2K\frac{r_1}{h}\left[1+\frac{J_1^2(x_{01})}{(\epsilon'_{\mathrm{unc}}-1)Q_S}\right]-\frac{1}{2}\left(\frac{1}{Q_{W0}}+\frac{3}{Q_{EX0}}\right)\right\}\approx(\epsilon'_{\mathrm{unc}}-1)\left(1+2K\frac{r_1}{h}\right) \quad (21)$$

$$\epsilon''=\epsilon''_{\mathrm{unc}}\left(1-2K\frac{r_1}{h}\right)+\frac{\epsilon'-1}{2}\left(\frac{1}{Q_{W0}}+\frac{3}{Q_{EX0}}\right) \quad (22)$$

where ϵ'_{unc} and $\epsilon''_{\mathrm{unc}}$ are the uncorrected value of ϵ' and ϵ'' calculated by means of (13) and (14); h is the relative height of cavity, $h=H/R$; and for an example, the correction constant for Fig. 1(a) configuration is

$$K=\sum_{n=1}^{\infty}\frac{A_n^2J_1^2(x_{on})}{x_{on}}\approx 0.327 \quad (23)$$

in which A_n is the amplitude of TM_{0n} mode in sample insertion hole [11].

In a practical example, these effects are shown in Fig. 6 and Fig. 7, respectively. Equations (21) and (22) can also be used in Fig. 1(b) configuration but the constant K will depend on dimensions of dielectric tube and the ϵ' of measured sample materials [11].

III. Experimental Results

Both values of ϵ' and ϵ'' were measured by means of active frequency techniques [1] in a narrow frequency band 2.0 to 2.23 GHz at 22°C; a large variety of test samples were tried out and the results of some measurements are given in Table I below.

The frequency stability of the cavity is approximately 2×10^{-6} and the measurement error in Q_L is ±1.25 percent ($Q_L\geqslant500$) and ±7 percent ($Q_L=200$) [1]. It means the measurement error is negligible in ϵ' and less than ±2 percent in ϵ''. A major error arises in the mechanical measurement of sample size. If a micrometer with an accuracy of ±0.001 in is used, for small sampling pipe, this error is approximately ±1 to ±2 percent and this leads to an error of ±2 to ±4 percent in ϵ'.

The measured results must be revised by the use of equations (21) and (22). After correction, an error of less than 1 percent in ϵ' and 5 percent in ϵ'' is expected. Comparing the measured value with the ones in the literature, it can be concluded that the small differences are

TABLE I
RESULTS OF MEASUREMENTS ON ϵ' AND ϵ'' USING EXACT FIELD CALCULATIONS, INCLUDING THE EFFECTS OF ALL NONIDEAL CONDITIONS. (f=2.23 GHz, T=22°C)

Material	R_1 (in mm)	R_2 (in mm)	$\frac{\delta f}{f_0} \times 10^3$	$\delta(\frac{1}{Q_L}) \times 10^3$	ε'_{unc}	ε''_{unc}	measured ε: ε'	measured ε: ε''	reference value: ε'	reference value: ε''	Ref.
Teflon	3.188		7.17	.012	2.00	$<10^{-3}$	2.09	$<10^{-3}$	2.10	3×10^{-4}	13,15
Quartz	2.337	3.321	10.29	.026	3.61	0.0032	3.80	0.002	3.78	2.3×10^{-4}	13
Pyrex	2.000	2.997	12.88	0.30	4.63	0.042	4.75	0.040	-	-	
Benzene	1.418	2.121	1.69	.014	2.16	0.0046	2.27	0.0038	2.28	.0028	13
Transformer Oil 10 C	2.591	2.756	5.15	.043	2.08	0.0044	2.18	0.0037	2.18	.0028-.006	13,15
Acetone	.688	.978	6.65	0.62	20.2	0.88	20.9	0.85			
Methanol	.304	.749	1.39	1.78	22.0	13.4	22.6	13.1	22-24	12.5-13.5	7
1 - Octanol	.684	.978	0.61	0.48	2.82	0.72	2.87	0.70			
1 - Propanol	.305	.756	0.21	0.48	4.14	3.66	4.19	3.63	3.7 -4.4	1.45-2.81	14
2 - Propanol	.699	.965	0.92	2.10	3.65	3.02	3.73	2.96			
1 - Buthanol	.691	.959	0.89	1.36	3.61	2.02	3.70	1.96	3.43-4.0	.87-1.64	14
Water	.304	.700	1.214	0.295	74.2	8.80	76.8	8.62	76-78	8-12.5	14,15,8
	.432	.708	2.461	0.563	74.1	8.19	77.4	8.03			

introduced mostly by the different purity of materials and the difference in operating frequencies and temperatures.

IV. CONCLUSION

An exact field theory for cylindrical TM_{010} cavity perturbation is presented in this paper. The theoretical solution for lossy media shows that both real and imaginary part of the complex permittivity are a complex function of measured resonant frequency shift and variation in loaded Q-factor. A further theoretical analyses deals with the effects of the sample insertion hole, coupling elements, and lossy cavity wall. Among these, the sample insertion hole provides a major effect on both measured ϵ' and ϵ''.

Experimental results obtained on various materials show that the active cavity technique [1] is suitable for a wide range of precise measurements. Excluding the mechanical measurement error of sample size, the measurement errors arising from the present theory and measurement system are less than 1 percent in ϵ' and 5 percent in ϵ''.

REFERENCES

[1] C. Akyel, R. G. Bosisio, and G. E. April, "An active frequency technique for precise measurements on dynamic microwave cavity perturbations," *IEEE Trans. Instrum. Meas.*, vol. IM-27, pp. 364–368, 1978.

[2] F. Horner *et al.*, "Resonance methods of dielectric measurements at centimeter wavelengths," *Proc. Inst. Elec. Eng.*, (London), vol. 93, pt. III, pp. 53–68, 1946.

[3] B. Agdur and B. Enander, "Resonance of a microwave cavity partially filled with a plasma," *J. Appl. Phys.*, vol. 33, pp. 575–581, 1962.

[4] D. Lukac, "The determination of electron density by means of a cylindrical TM_{010} microwave cavity," *Brit. J. Appl. Phys.*, vol. 1, pp. 1495–1499, 1968.

[5] M. A. Rzepecka, S. S. Stuchly, and M. A. K. Hamid, "Modified infinite sample method for routine permittivity measurements at microwave frequencies," *IEEE Trans. Instrum. Meas.*, vol. IM-22, pp. 41–46, 1973.

[6] S. S. Stuchly, M. A. Stuchly, and B. Carrado, "Permittivity measurements in a resonator terminated by an infinite sample," *IEEE Trans. Instrum. Meas.*, vol. IM-27, pp. 436–439, 1978.

[7] H. E. Bussey, "Dielectric measurements in a shielded open circuit coaxial line," *IEEE Trans. Instrum. Meas.*, vol. MI-29, pp. 120–124, 1980.

[8] E. C. Burdette, F. L. Cain, and J. Seals, "In vivo probe measurement technique for determining dielectric properties at VHF through microwave frequencies," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 414–427, 1980.

[9] A. J. Estin and H. E. Bussey, "Errors in dielectric measurements due to a sample insertion hole in a cavity," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-8, pp. 650–653, 1960.

[10] W. Meyer, "Dielectric measurements on polymeric materials by using superconducting microwave resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 1092–1099, 1977.

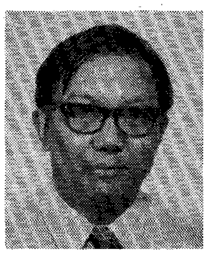
[11] S. H. Li and R. G. Bosisio, "Effects of sample insertion hole on the measurement of dielectric constant," to be published.

[12] J. P. Montgomery, "On the complete eigenvalue solution of ridged waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 547–555, 1971.

[13] A. R. von Hippel, *Dielectric Material and Application*. Cambridge, MA: M.I.T. Press, 1954, pp. 303–370.

[14] B. Terselius, and B. Ranby, "Cavity perturbation measurements of dielectric properties of vulcanizing rubber and polyethylene compounds," *J. Microwave Power*, vol. 13, pp. 327–334, 1978.

[15] R. F. Harrington, *Time Harmonic Electromagnetic Field*, New York: McGraw-Hill, 1961.



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Resonant Frequency Stability of the Dielectric Resonator on a Dielectric Substrate

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***Abstract*—A simple approximate method for predicting the resonant frequencies of TE modes of dielectric resonators is developed. By using this method, an analytical expression is derived for the resonant frequency stability of the dielectric resonator on a dielectric substrate, and the effect of the substrate on the stability is studied. The result is useful when the high-frequency stability is required.**

I. Introduction

DIELECTRIC resonators exhibiting high Q factors and very low temperature dependence of the resonant frequency have been recently developed [1]–[3]. They promise to shrink the size and cost of waveguide cavities. Also, they are useful elements of MIC's, and have been applied for filters [4] and oscillators [5]–[7].

A dielectric resonator structure commonly used in practical MIC's is that composed of a cylindrical dielectric sample placed on a dielectric substrate, one side of which is metallized as a ground plane, and of a metal tuning screw placed above the dielectric resonator sample. On the calculation of such a dielectric resonator structure, several works have been reported [8], [9]. However, none of them has described the resonant frequency stability. The degree of effect of the factors affecting the resonant frequency stability must be considered when the high stability is required especially in a local oscillator application [7].

The purpose of the present paper is to derive the analytical formula for the resonant frequency stability of the TE mode dielectric resonator on a dielectric substrate, and to show how the substrate affects the frequency stability. The result is useful in determining the temperature coefficient of the dielectric resonator material to realize the high-frequency stability in a practical MIC application.

II. Approximate Resonant Frequency

The resonant structure under consideration is shown schematically in Fig. 1. D is the diameter, and L the height of the dielectric resonator. ϵ_1, ϵ_2, and ϵ_r are the relative

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